**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans :-**

**In the problem given *μ* = 45 , *σ* = 8 , the work begins 10 mins after the car is**

**dropped , the time left to complete work is 50 mins . Probability that service manager**

**cannot meet his commitment = P(x>50) = 1- P(x<=50)**

**Here x is the time taken to complete the work .**

**Convert to 50 to z- score**

**Standard normal variable Z = (X- *μ*)/*σ***

**=(x-45)/8**

**= (50-45)8**

**= 0.625**

**Z -calculated value = 0.625**

**Then in the z -table the value is 0.73232**

**PR=(z<=0.625) = 0.7323 = 73.237%**

**Probability that service manager will not meet his commitment is 100-73.237 = 0.2676**

**So the answer is B. 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans : -**

1. **FALSE**

**The probability of age > 44 (which corresponds to P(Z > 1)) is approximately 0.1587.**

**The probability of age between 38 and 44 (which corresponds to P(0 < Z < 1) ) is approximately 0.3413.**

**This means that there are more employees with ages between 38 and 44 than there are employees with ages greater than 44.**

1. **TRUE**

**Since the normal distribution is continuous, we can interpret this probability as the proportion of employees under the age of 30. we can multiply probability by the total number of employees (400).**

**Number of employees under 30 = 0.0912 × 400 ≈ 36.48**

**So, about 36 employees would be expected to be under the age of 30.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans :-**

**1. (2X1):**

**- Distribution: Normal N(2μ, 4σ^2)**

**- Parameters: Mean = 2μ, Variance = 4σ^2**

**2. (X1 + X2):**

**- Distribution: Normal N(2μ, 2σ^2)**

**- Parameters: Mean = 2μ, Variance = 2σ^2**

**Both random variables (2X1) and (X1 + X2) follow normal distributions, but they have different variances despite having the same mean. The difference in their variances indicates that the spread (dispersion) of their values is distinct, with (2X1) being more spread out than (X1 + X2). This can be seen from the fact that (2X1) has a variance of (4σ^2) while (X1 + X2) has a variance of (2σ^2)**.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans :-**

**Probability of getting value between a and b is 0.99.**

**So , the probability getting outside value a and b is = 1-0.99 = 0.01**

**Probability towards left of a = -0.01/2 = -0.05**

**Probability towards right of b = 0.01/2 = 0.05**

**By finding the Standard Normal Variable (z) , need to calculate X:**

**Z = (X-*μ*)/ σ**

**For a probability of 0.005 , z value is -2.57**

**Z\* σ+ μ=x**

**-(-2.57)\*20+100=151.4**

**(-2.57)\*20+100=48**

**Option D is correct answer**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans :- Range is Rs (99.008, 980.992) in Millions**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans :- 5th percentile of profit (in Million Rupees) is 170.0**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans :- Probability of division 1 making a loss P(0.0478) is higher than division 2 P(0.0400)**